**ECE 4705 Lab**

**Experiment 1 – Spectrum of Pulse Train**

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**ECE4705L\_02**

**ECE 4705L EXPERIMENT 1**

**SPECTRUM OF PULSE TRAIN[[1]](#footnote-1)**

# INTRODUCTION

1. Background Theory
   1. Basic Definition of a Pulse Train

Pulse trains are used to describe a periodic nonsinusoidal waveform, such as square waves, triangle waves and sawtooth waves to name a few. These waveforms can be represented by functions in the time domain f(t) as well as in the frequency domain F(ω). When there is an infinite periodic signal, it can be described as an exponential Fourier Series. For any given pulse train in the time domain we have the function,

𝑓(𝑡) = ∑∞*n=-∞*𝐹*ne jnωot*

where the value Fn is the signal in the frequency domain.

Chart, bar chart, histogram

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Figure 1.1 – Example of Rectangular Pulse Train with Pulse Width τ.

* 1. Derivation of Pulse Train Spectra

To further examine the behavior of a pulse train you must first perform a Fourier transform by taking the integral

𝐹(𝜔) =

where, in the case of a rectangular pulse train, f(t) has a constant magnitude we will call A, and, because f(t) has constant value only throughout the duration of the pulse width gives us,

**𝐹(𝜔) =**

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=A picture containing text, watch

Description automatically generated

=Diagram

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**𝐹(𝜔)** =A picture containing logo

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where τ is the pulse width in relation to the total period, also known as the duty cycle. Given certain paramters for a given signal, performing this transform of f(t) results in an equation that can be used to describe any signal in the frequency domain. From this equation, magnitude and phase spectrum plots can be graphed to better show the characteristics of the pulse. An example using a rectangular pulse at a given frequency 𝜔 is shown below:

Chart, line chart

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Figure 1.2 – Magnitude of Rectangular Pulse in Frequency Domain

Chart, line chart

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Figure 1.3 – Phase of Rectangular Pulse in Frequency Domain

Using what are known as Fourier Transforms to analyze signals is useful when working with electrical AC components, most of which function specific at frequencies, because it allows an analog signal which, normally exists in the time domain, to be represented in the frequency domain. If we want to solve for the coefficients of the magnitude and phase, we can manipulate 𝐹(𝜔) to get,

𝐹(𝜔) = Ad*sinc*(nd)

– where τ = d and = f – to create a spectrum plot that describes a signal at the harmonic frequencies. By knowing a signal’s fundamental frequency, we can find the coefficients at each nth harmonic frequency.

**OBJECTIVE**

The objective of this experiment is to become familiar with the Fourier frequency spectrums and the manner in which the amplitude is obtained in the laboratory.

# EQUIPMENT

HP Spectrum Analyzer, Oscilloscope, HP 3312A Function Generator (or available digital function generator)

# PRE-LAB & DISCUSSION

1. Compute and plot the two-sided amplitude spectrum and two-sided phase spectrum for

(a) a periodic rectangular pulse train with amplitude 100 mV, period 10 µs (freq=100kHz) and duty cycle, in the range, .

(b) a periodic rectangular pulse train with amplitude 100 mV, period 10 µs (freq=100kHz) and duty cycle, in the range, .

(c) a periodic rectangular pulse train with amplitude 100 mV, period 10 µs (freq=100kHz) and duty cycle. , in the range, .

(d) a periodic triangular pulse train with amplitude 100 mV, period 10 µs (freq=100kHz) , in the range, . Note: duty cycle of the rectangular pulse is =.

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Figure 1.4 – Magnitude Spectrum of Rectangular Pulse with Duty Cycle 1/2



Figure 1.5 – Phase Spectrum of Rectangular Pulse with Duty Cycle ½

When examining the magnitude spectrum shown in figure 1.4, the maximum value, for any given frequency up until 1.5MHz, that can be achieved for this particular rectangular pulse is 0.05. The phase spectrum shows us that every nth minus 1 harmonic frequency there is a phase spike corresponding to the positive magnitudes of the Fourier series corresponding to the rectangular pulse.

1b. A piece of paper with writing

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Figure 1.6 – Magnitude Spectrum of Rectangular Pulse with Duty Cycle 1/5



Figure 1.7 – Phase Spectrum of Rectangular Pulse with Duty Cycle 1/5

A common pattern to take note off is that, for a given rectangular pulse with duty cycle 1/n, the magnitude of the pulse drops to zero every nth harmonic frequency.

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Figure 1.8 – Magnitude Spectrum of Rectangular Pulse with Duty Cycle 1/7



Figure 1.9 – Phase Spectrum of Rectangular Pulse with Duty Cycle 1/7

An interesting observation to take note of is that the narrower the pulse of a pulse train is in the time domain, the wider it appears in the frequency domain. The faster the change in value occurs on the pulse train, the higher the frequencies will be needed to create that change, which for a rectangular pulse train is higher given a smaller duty cycle.

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Figure 1.10 – Magnitude Spectrum of Triangular Pulse



Figure 1.11 – Phase Spectrum of Triangular Pulse

The reason a triangle wave has such a narrow magnitude spectrum because most of the magnitudes of the spectrum of the wave are covered within the first harmonics. It makes sense given the fact that the triangle pulse train does not have nearly as much discontinuity as the rectangular pulse trains.

**CONCLUSION**

This lab introduced us to the analysis of signals represented by Fourier series. By performing Fourier transforms on these signals, we can come up with an equation that can be used to see how the signal behaves in the frequency domain, as opposed to the time domain. Using Matlab, we were able to compute and plot the magnitude and phase spectra of the given signal. We noted that the skinnier a pulse’s magnitude spectrum, the wider the phase spectrum became as well, and these changes were both related to the duty cycle of the pulse. The smaller duty cycle for a pulse, the skinnier the magnitude spectrum, and so, a wider phase spectrum is also a result of this smaller duty cycle.

Appendix A

1. MATLAB code:

clear;clc;

T=10e-6;

f0=1/T;

n=-15:1:15;

f=n\*f0;

a1=100e-3;

duty1=1/2;

cn1=a1\*duty1\*sinc(n\*duty1);

threshold=-1.94909e-18;

cn1(cn1>threshold&cn1<0)=0;

figure(1);

stem(f,abs(cn1));

title('Magnitude Spectrum plot');

xlabel('frequency (Hz)');

ylabel('Magnitude C(n)');

grid on;

figure (2);

stem(f,angle(cn1));

title('Phase Spectrum plot');

xlabel('frequency (Hz)');

ylabel('Phase C(n)');

grid on;

duty2=1/5;

cn2=a1\*duty2\*sinc(n\*duty2);

cn2(cn2>threshold&cn2<0)=0;

figure(3);

stem(f,abs(cn2));

title('Magnitude Spectrum plot');

xlabel('frequency (Hz)');

ylabel('Magnitude C(n)');

grid on;

figure (4);

stem(f,angle(cn2));

title('Phase Spectrum plot');

xlabel('frequency (Hz)');

ylabel('Phase C(n)');

grid on;

duty3=1/7;

cn3=a1\*duty3\*sinc(n\*duty3);

cn3(cn3>threshold&cn3<0)=0;

figure(5);

stem(f,abs(cn3));

title('Magnitude Spectrum plot');

xlabel('frequency (Hz)');

ylabel('Magnitude C(n)');

grid on;

figure (6);

stem(f,angle(cn3));

title('Phase Spectrum plot');

xlabel('frequency (kHz)');

ylabel('Phase C(n)');

grid on;

cn4=50e-3\*(sinc(n/2)).^2;

figure(7);

stem(f,abs(cn4));

title('Magnitude Spectrum plot');

xlabel('frequency (Hz)');

ylabel('Magnitude C(n)');

grid on;

figure(8);

stem(f,angle(cn4));

title('Phase Spectrum plot');

xlabel('frequency (Hz)');

ylabel('Phase C(n)');

grid on;

1. Based on a lab from Dr. James Kang [↑](#footnote-ref-1)